Vanishing environment-induced decoherence

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For a central system uniformly coupled to a XY spin-1/2 bath in a transverse field, we explicitly calculate the Loschmidt echo(LE) to characterize decoherence quantitatively. We find that the anisotropy parameter γ affects decoherence of the central system sensitively when $\gamma \in [0, 1]$, in particular, the LE becomes unit without varying with time when $\gamma = 0$, implying that environment-induced decoherence vanishes . Some other cases in which the LE is unit are discussed also.

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I. INTRODUCTION

Coherence of a quantum state is very fragile because of the existence of its environmental degrees of freedom coupled to it, which has become the major obstacle in constructing quantum computer [1, 2]. To protect the quantum information, we generally use the quantum error correction scheme which can correct the quantum errors to protect the encoded quantum states[3, 4, 5]; We can also use the scheme to find the decoherence-free subspaces and some other schemes to protect the quantum information[6, 7]. Generally we need several physical qubits to realize one logic qubit in these schemes. It will be very interesing if we can find a quantum systems in which the quantum states can be naturally protected.

On the other hand, many physicists took attention to the relationship among the concepts of environment, decoherence, and irreversibility, these investigations may provide new perspective of how to overcome decoherence and renewed understanding for the crossover between quantum and classical behavior [8]. In the study of quantum-classical transition in quantum chaos, the concept of Loschmidt echo(LE) was introduced[9], and we will use it to determine decoherence of a central system.

With the development of quantum information, the concept of entanglement (concurrence) was used to investigate the quantum phase transition(QPT) [11, 12]. Recently, Carollo and Pachos [13] have established the relation between geometric phases(GP) and criticality of spin chain, Zhu [14] analyzed the scaling of the geometric phases, and Hamma [15] found that a non-contractible GP of the ground state is also a witness of QPT. Quan *et al* [16] found that the decay of the LE is enhanced by quantum criticality in Ising spin-1/2 model. These particular features in the XY model above seem to imply that there exist distinct properties with respect to decoherence of a central quantum system surrounded by it. In this paper, we show that the anisotropy parameter γ affects decoherence of the central system sensitively when $\gamma \in [0, 1]$, in particular, the LE becomes unit without varying with time when $\gamma = 0$, implying that environment-induced decoherence vanishes . Some other cases in which the LE is unit are discussed also.

II. DERIVATION OF THE LOSCHMIDT ECHO FOR A CENTRAL SYSTEM

Firstly, we analyze the XY model as a starting point, since it is exactly solvable and presents a rich structure. The system-bath model can be described by the Hamiltonian $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_E + \mathcal{H}_{SE}$, the central (two-state) system, characterized by the ground state $|0\rangle$ and the excited state $|1\rangle$, has a free Hamiltonian $\mathcal{H}_S = w_e |1\rangle \langle 1|$ and is coupled to all spins in the bath through the interaction $\mathcal{H}_{SE} = -\delta \sum_l |1\rangle \langle 1| \sigma_l^z$, where δ represents the coupling constant. Our model differs from the model in [17] where the system only interacts with the first spin in the bath. The chain in a transverse field has nearest neighbor interactions with Hamiltonian expressed by

$$\mathcal{H}_E = -\sum_{l=-M}^{M} \left(\frac{1+\gamma}{2} \sigma_l^x \sigma_{l+1}^x + \frac{1-\gamma}{2} \sigma_l^y \sigma_{l+1}^y + \lambda \sigma_l^z \right), \tag{1}$$

where $M = (N-1)/2$ for N odd, and the operators σ_l^α ($\alpha = x, y, z$) are the usual Pauli operators defined on the l-th site of the lattice. The constants $\gamma \in [0, +\infty]$ and $\lambda \in \mathbb{R}$ represent the anisotropy parameter in the next-neighbor spin-spin interaction and an external magnetic field. The model defined by Eq.(1) has a rich structure [18], i.e., when the anisotropy parameter γ is set to $(0, 1]$, the model of Eq.(1) belongs to the Ising universality class which has a critical point only at $\lambda_c = 1$, however, when $\gamma = 0$, it belongs to the XY universality class and the critical region is $\lambda_c \in (-1, 1)$.

We assume the central system to be prepared in a superposition state $|\Psi_S\rangle = \alpha |0\rangle + \beta |1\rangle$, thus the initial system-environment state can be written as $|\Psi_{SE}(0)\rangle = (\alpha |0\rangle + \beta |1\rangle) |\Psi_E(0)\rangle$. From the evolved reduced density matrix of the system $\rho_S(t) = \mathtt{Tr}_E |\Psi_{SE}(t)\rangle \langle \Psi_{SE}(t)|$, we obtain

$$\begin{aligned} \rho_S(t) &= |\alpha|^2 |0\rangle \langle 0| + \alpha\beta^* R(t) |0\rangle \langle 1| \\ &\quad + \alpha^* \beta R^*(t) |1\rangle \langle 0| + \beta^2 |1\rangle \langle 1| . \end{aligned} \tag{2}$$

Clearly, in the basis of the eigenstates $|0\rangle$ and $|1\rangle$, the diagonal terms in Eq.(2) do not evolve with time, and only the off-diagonal terms will be modulated by the decoherence factor $R(t)$, which is the overlap between two states of the environment obtained by evolving the same initial state $|\Psi_E(0)\rangle$ driven by two different effective Hamiltonians $\mathcal{H}_0$ and $\mathcal{H}_1$. As discussed in [16], for the model (1) we have $\mathcal{H}_j = -\sum_{l=-M}^{M}\left[\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + (\lambda + j\delta)\sigma_l^z\right]$ with $j = 0$ or 1. $R(t)$ can be defined as

$$R(t) = \langle\Psi_E(0)|\, e^{it\mathcal{H}_0}e^{-it\mathcal{H}_1}\,|\Psi_E(0)\rangle\,, \tag{3}$$

while the LE is determined from $L(t) = |R(t)|^2$, which is also called fidelity. If the initial surrounding environment is prepared in the ground state of $\mathcal{H}_0$, i.e., $|\Phi_0\rangle$, Eq.(3) will reduce to a simpler form

$$R(t) = \langle\Phi_0|\, e^{-it\mathcal{H}_1}\,|\Phi_0\rangle\,, \tag{4}$$

where an irrelevant phase factor is removed.

Next we will deduce the detailed expression of $R(t)$ for the model (1). In the standard way, the two Hamiltonians $\mathcal{H}_j$ can be diagonalized in terms of a suitable set of fermionic creation and annihilation operators $\mu_k^{(j)}$ as

$$\mathcal{H}_j = \sum_{k=1}^{M}\varepsilon_k^{(j)}\left[\mu_k^{(j)\dagger}\mu_k^{(j)} - 1\right]. \tag{5}$$

When getting the equation above, we have applied to each spin a rotation of ϕ around the z direction $\mathcal{H}_j(\phi) = g(\phi)\mathcal{H}_j g^\dagger(\phi)$ with $g(\phi) = \prod_{l=-M}^{M}\exp(i\sigma_l^z\phi/2)$, the Jordan-Wigner transformation mapping the spins to one-dimensional spinless fermions with creation and annihilation operators a_l and $a_l^\dagger$ via the relation $a_l = \left(\prod_{i<l}\sigma_i^z\right)(\sigma_l^x + i\sigma_l^y)/2$, and the Fourier transformation of the fermionic operators described by $c_k = \left(1/\sqrt{N}\right)\sum_l a_l\exp(-i2\pi lk/N)$. The energy spectrum in Eq.(5) is

$$\varepsilon_k^{(j)} = \sqrt{\left[\cos\left(\frac{2\pi k}{N}\right) - (\lambda + j\delta)\right]^2 + \gamma^2\sin^2\left(\frac{2\pi k}{N}\right)}, \tag{6}$$

and through a Bogliubov transformation the operators appearing in the Hamiltonians $\mathcal{H}_j$ we have

$$\mu_k^{(j)} = c_k\cos\left[\frac{\theta_k^{(j)}}{2}\right] - ic_{-k}^\dagger e^{2i\phi}\sin\left[\frac{\theta_k^{(j)}}{2}\right], \tag{7}$$

where the angles $\theta_k^{(j)}$ is the Bogliubov coefficients satisfying the equation

$$\cos\left[\theta_k^{(j)}\right] = \frac{\cos\left(\frac{2\pi k}{N}\right) - (\lambda + j\delta)}{\varepsilon_k^{(j)}}. \tag{8}$$

It is easy to check that the spinless Fermion operators $\mu_{\pm k}^{(j)}$ satisfy

$$\mu_{\pm k}^{(0)} = \mu_{\pm k}^{(1)}\cos(\theta_k) \mp i\mu_{\mp k}^{(1)\dagger}e^{2i\phi}\sin(\theta_k)\,, \tag{9}$$

where $\theta_k = \left[\theta_k^{(0)} - \theta_k^{(1)}\right]/2$.

According to Eq.(9), the ground state of $\mathcal{H}_0$ can be expressed as

$$|\Phi_0\rangle_{XY} = \prod_{k=1}^{M}\left[\cos(\theta_k)\,|0\rangle_k\,|0\rangle_{-k} + ie^{2i\phi}\sin(\theta_k)\,|1\rangle_k\,|1\rangle_{-k}\right], \tag{10}$$

for any opetators $\mu_{\pm k}^{(0)}$ we have $\mu_{\pm k}^{(0)}|\Psi_0\rangle = 0$. $|0\rangle_k$ and $|1\rangle_k$ are the vacuum and single excitation of the kth mode, $\mu_k^{(1)}$, respectively.

As expected that the ground state of $\mathcal{H}_0$ is taken as the initial surrounding environment state, after some algebraic manipulations the decoherence factor is obtained

$$R(t) = \prod_{k=1}^{M}R_k(t) = \prod_{k=1}^{M}\left[\sin^2(\theta_k) + \cos^2(\theta_k)e^{i2\varepsilon_k^{(1)}t}\right], \tag{11}$$

so we can express the LE as

$$L(t) = |R(t)|^2 = \prod_{k=1}^{M}[1 - \sin^2(2\theta_k)\sin^2(\epsilon_k^{(1)}t)]. \tag{12}$$

The term $R_k(t) \equiv \sin^2(\theta_k) + \cos^2(\theta_k)e^{i2\varepsilon_k^{(1)}t}$ is a decoherence factor for the k-th mode, and its modulus square is always not larger than one. It is interesting to mention that the Berry phase of the ground state in the XY model is of sum form for each mode [13, 14], while this decoherence factor (11) is of multiplying form for each mode. Furthermore, Eq.(11) is analogous to the one for non-interacting spin environments[20] and Cucchietti generalized Quan's results[21] .

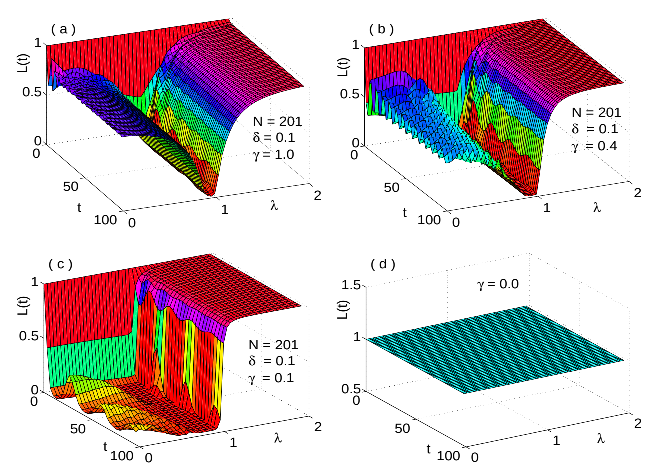

FIG. 1: Three dimensional diagram of the $L(t)$ as a function of t and λ with $N = 201$ and $\delta = 0.1$. With decreasing γ from 1.0 to 0.1, (a)-(c) show that the range of λ where the decay of $L(t)$ is enhanced increases. (d) shows that when $\gamma = 0.0$, the $L(t)$ is unit always, regardless of what N, λ and δ are.

To better understand the LE (12), we plot it as a function of t and λ in Fig.1, and as a function of only t in Fig.2. For simplicity, we only set $N = 201$ and $\delta = 0.1$. It is demonstrated that the decay of $L(t)$ is enhanced at the critical point of quantum phase transition $\lambda_c = 1$ in Fig.1(a), since the XY model with $\gamma = 1$ corresponds to the Ising model. There exists a deep valley in the around the line $\lambda = 0.9$, which is the same results as [16]. However, for the general XY model where γ is adjusted in $(0, 1)$, we find that the decay of $L(t)$ is enhanced in a different degree in the range $\lambda \in (0, 1)$. When $\gamma = 0.4$, the applitudes of $L(t)$ in Fig.1(b) is smaller than those corresponding to Fig.1(a) and the range of λ resulting in $L(t) = 0$ increases. When we continue to decrease γ to 0.1 in Fig.1(c), it is seen that the $L(t)$ nearly approaches zero in the range $\lambda \in (0, 1)$, where the central system transits from a pure state to a mixed state. So we can conclude that for a smaller γ, the critical point of quantum phase transition is the transition point of whether the decay of the $L(t)$ is enhanced or not.

Comparing with the results in [16], for the general XY model, we also see that $L(t)$ decays and revives as time increases in Fig.2(e) and (f). This may serve as a witness of QPT. At the same time, if we appropriately adjust the parameters N, δ and λ as shown in Fig.2(e) and (f), it is found that the two plots of $L(t)$ with the same γ have the identical profile, indicating that the period of the revival of the $L(t)$ is proportional to the size of the surrounding system in the case of finite N. Fig.2(g) and (h) reflect that the decreasing γ leads to fast decaying of the $L(t)$, which complies with the situation described by Fig.1(a)-(c). In quantum chaos[9] the sensitivity of perturbations in the Hamiltonian system can be understood according to the LE[22]. Here, for some paremeters shown in Fig.2(g) and (h), the $L(t)$ becomes chaotic, which is due to the competition between the two phases separated by $\lambda_c = 1$.

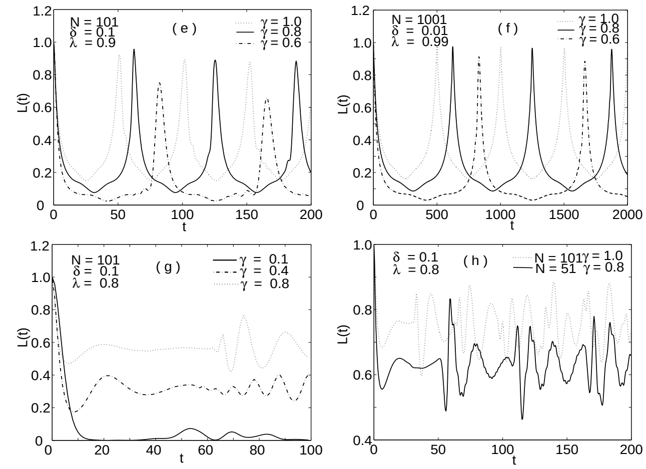

FIG. 2: The $L(t)$ as a function of t. (e) and (f) show that with decreasing γ the quasiperiod of the $L(t)$ increases, and it is proportional to the size of the surrounding system. The quasiperiod stems from quantum phase transiton at $\lambda_c = \lambda + \delta$=1. (g) shows that with decreasing γ the decay of the $L(t)$ is enhanced faster. (h) shows that for some parameters λ (away from critical point $\lambda_c = 1$) the $L(t)$ becomes chaotic, which is due to the competition between the two phases separated by $\lambda_c = 1$.

III. ANALYSIS OF VANISHING DECOHERENCE

Interestingly, we find that the $L(t)$ does not vary with time in the XX model with $\gamma = 0$, i.e., the coherence of the central spin will not be affected by the special environment. The reasons are in the following. From Eqs.(6-8), we see that

$$\lim_{\gamma\to 0} \cos\left[\theta_k^{(j)}\right] = \pm 1, \tag{13}$$

which directly results in that the ground state of $\mathcal{H}_0$ no longer lies in the two-dimensional Hilbert space spanned by $|0\rangle_k |0\rangle_{-k}$ and $|1\rangle_k |1\rangle_{-k}$, but only one of them like Eq.(14). To obtain the explicit form of the ground state, we let $\cos(2\pi k_1/N) = \lambda$ and $\cos(2\pi k_0/N) = \lambda + \delta$ in Eq.(8), and from them know that $k_0 < k_1$. Considering Eqs.(8-10), we can express the ground state

$$|\Phi_0\rangle_{XX} = \prod_{k=1}^{k_0} |0\rangle_k |0\rangle_{-k} \prod_{k=k_0+1}^{M} (ie^{2i\phi}) |1\rangle_k |1\rangle_{-k}, \tag{14}$$

since $\cos(\theta_k) = 1$ in Eq.(10) when $k < k_0$, and $\mu_k^{(0)} = \mu_k^{(1)}$ $(k < k_0)$ or $-\mu_k^{(1)}$ $(k > k_0)$. Therefore the LE becomes

$$L(t) = \lim_{\gamma\to 0} |R(t)|^2 = 1, \tag{15}$$

which means that the $L(t)$, initial equal to 1, will not decay at all, and the central system preserves its initial coherence except for an additional phase factor in one of eigenstate of the central system. For our model with the central system surrounded by the XX spin-1/2 bath, notice that the purity [16, 19] $P = \mathrm{Tr}_S(\rho_S^2)$ is defined to describe decoherence, it is $P = 1 - 2|\alpha\beta|^2 [1 - L(t)] = 1$ also, independent of the central system and this special environment. What's more, the result (15) is regardless of the number N of the lattice and what the external magnetic field λ is, it seems to be counterintuitive, but it is indeed the case. We can see Fig.1(d). At the same time, we emphasize that the non-decay of the LE in this case is nontrivial because of the difference between $\mathcal{H}_0$ and $\mathcal{H}_1$.

The result can be better understood as follows. Evolving from the initial system-environment state $|\Psi_{SE}(0)\rangle = (\alpha|0\rangle + \beta|1\rangle)|\Psi_E(0)\rangle$, which are not entangled, it becomes $|\Psi_{SE}(t)\rangle = \alpha|0\rangle|\Phi_0(t)\rangle + \beta|1\rangle|\Phi_1(t)\rangle$ at an arbitrary t, where $|\Phi_0(t)\rangle$ and $|\Phi_1(t)\rangle$ are driven by the Hamiltonians $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. It is known that

$\mathcal{H}_0$ and $\mathcal{H}_1$ are different, however, it happens that the anisotropy parameter $\gamma = 0$ leads to their same evolution, resulting in $\langle\Phi_0(t)|\Phi_1(t)\rangle = \exp(i\varphi)$. The real time-dependent φ denotes an additional phase factor.

Finally, we assume that the XX model environment to be initially prepared in an arbitrary excited state with $\gamma = 0$. A n-particle state has the form $\mu_{k_1}^{(0)\dagger}\mu_{k_2}^{(0)\dagger}\ldots\mu_{k_n}^{(0)\dagger}|\Phi_0\rangle_{XX}$, with all the k_i distinct, i.e., it is

$$\begin{aligned}|\Phi_n\rangle_{XX} &= \prod_{k'=k_1}^{k_n}|1\rangle_{k'}|0\rangle_{-k'}\prod_{k=1}^{k_0}|0\rangle_k|0\rangle_{-k}\\ &\times\prod_{k=k_0+1}^{M}(ie^{2i\phi})|1\rangle_k|1\rangle_{-k}\,, \qquad (16)\end{aligned}$$

where $k \neq k'$. Substituting Eq.(16) into Eq.(4), we find that the LE is $L(t) = 1$ also, which implies that the partial excited states of the environment does not induce decoherence to the central system. However, if the environment is initially prepared in a thermal state, the LE is no longer equal to unit, but will decay with time.

Of particular interest is the case in which the XY model lies initially in an excited state. The m-particle state can be written as

$$\begin{aligned}|\Phi_m\rangle_{XY} &= \prod_{k'=k_1}^{k_m}|1\rangle_{k'}|0\rangle_{-k'}\prod_{k=1,k\neq k'}^{M}[\cos(\theta_k)|0\rangle_k|0\rangle_{-k}\\ &+ie^{2i\phi}\sin(\theta_k)|1\rangle_k|1\rangle_{-k}]. \qquad (17)\end{aligned}$$

After calculation by substituting Eq.(17) into Eq.(4) the LE is derived as $L(t) = \prod_{k=1}^{M}\left[1-\sin^2(2\theta_k)\sin^2(\varepsilon_k^{(1)}t)\right]$ with $k \neq k'$. It can be seen that: (i) the excited states $\prod_{k'=k_1}^{k_m}|1\rangle_{k'}|0\rangle_{-k'}$ have no any contributions to modulating the LE; (ii) if all the particles are excited, i.e., $|\Phi_m\rangle_{XY} = \prod_{k=1}^{M}|1\rangle_k|0\rangle_{-k}$, which is assumed to be the initial state of the bath, the LE of the central system is unit also. Notice that if the initial state of the XY model bath is thermal, the LE of the central system will decay with time. The vanishing decoherence may arise in view of the non-interacting environments in [20]: if we let all spins lie initially in either up or down, the decoherence factor will be unit as well. It is worthwhile for us to find out its physical nature.

IV. CONCLUSIONS

In summary, for a central system uniformly coupled to a XY spin-1/2 bath in a transverse field, we explicitly calculate the Loschmidt echo(LE) to characterize decoherence quantitatively. We find that the anisotropy parameter γ affects decoherence of the central system sensitively when $\gamma \in [0, 1]$, in particular, the LE becomes unit without varying with time when $\gamma = 0$, implying that environment-induced decoherence vanishes. At the same time, we show that decoherence can vanish provided that the initial state of the XX spin-1/2 bath lies in either the ground state or the partial particles are excited, or it lies in the state that all particles are excited. Although it is difficult to make the initial state of the spin bath pure at zero temperature and then difficult to fulfil the vanishing decoherence in real experiments, in a theoretical sense, our findings may shed light on understanding of decoherence.

V. ACKNOWLEDGEMENT

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